# Atwood Machine Lab Report

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Lab Section 12

TA:

# 1 ) Introduction

An Atwood machine shows the relationship between forces and acceleration. The masses of both weights can be measured and the forces can be calculated from the measured masses and gravity. By varying the weights and measuring acceleration, the relationship between forces and acceleration can be calculated.

# 2 ) Theory

An Atwood machine consists of two weights ($m_1$ and $m_2$) connected by a string ($S$). The string is placed on a wheel that allows the weights to move up and down. The system can be modeled with the two free body diagrams:

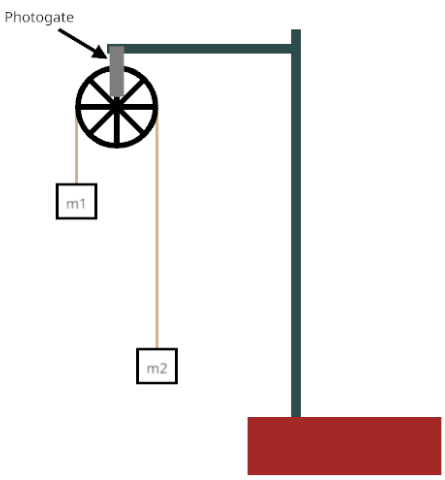
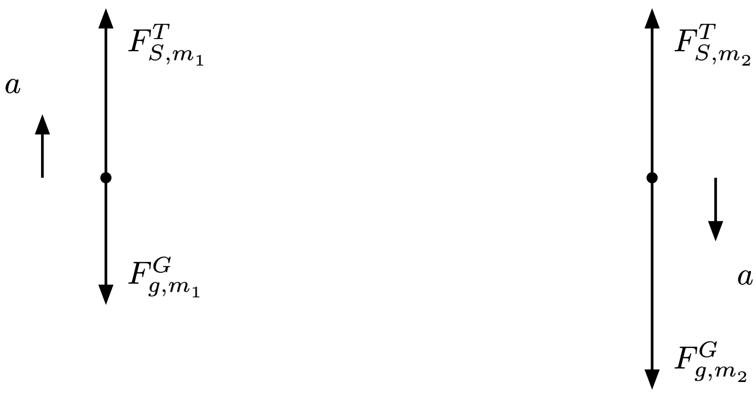



Figure 1: A model of the Atwood machine used in the procedure




The sum of forces in the y direction for each weight can be found by adding the two forces in each diagram. Since these are the only forces acting on the weights

$$\sum F_{y,m_1} = F^T_{S,m_1} + F^G_{g,m_1} \quad 2.1$$

$$\sum F_{y,m_1} = F^T_{S,m_1} + F^G_{g,m_1} \quad 2.2$$

Taking the downward direction to be positive, $F^G_{g,m_1}$ and $F^G_{g,m_2}$ can be found with the equation t

$$F = ma \tag{2.3}$$

$$F^G_{g,m_1} = m_1 g \tag{2.4}$$

$$F^G_{g,m_2} = m_2 g \tag{2.5}$$

Assuming that the string is not stretching, $m_1$ and $m_2$ are each exerting equal forces on each of the weights

$$F^T_{S,m_1} = F^T_{S,m_2} = F^T \tag{2.6}$$

Since the rope is not stretching, the objects are accelerating with the same magnitude but in opposite directions. Using this fact, the values found in Equation 2.4, Equation 2.5, and Equation 2.6, and that $\sum F_y = ma_y$ the equations can be simplified to:

$$m_1 a = F^T + m_1 g \tag{2.7}$$

$$-m_2 a = F^T + m_2 g \tag{2.8}$$

The first equation can now be solved for $F^T$ and can be plugged into the second equation

$$F^T = m_1 a - m_1 g \tag{2.9}$$

$$-m_2 a = m_2 g - (m_1 a - m_1 g) \tag{2.10}$$

$$-m_2 a = m_2 g - m_1 a + m_1 g \tag{2.11}$$

Isolating $a$ then gives an equation for acceleration in terms of $m_1$ and $m_2$

$$m_1 a - m_2 a = m_2 g + m_1 g \tag{2.12}$$

$$a = \frac{m_2 g + m_1 g}{m_1 - m_2} \tag{2.13}$$

Pulling $g$ out of the right side of the equation gives

$$a = g\left(\frac{m_2 + m_1}{m_1 - m_2}\right) \tag{2.14}$$

Using $M$ to represent $\frac{m_2+m_1}{m_1-m_2}$, the equation used for this procedure is found:

$$a = gM \tag{2.15}$$

## 3 ) Procedure

An Atwood Machine was created by suspending a string from a wheel attached to a lab support. A photogate so that it was blocked multiple times while the wheel spun. On each end of the string, weights were attached of varying masses.

The experiment consisted of 8 trials. The first 6 trials were calculated with varying weights for $m_1$ and $m_2 = m_1 + 0.005\text{kg}$ The value of $M$ was calculated for each trial The weight was held up until the PASCO Capstone software was recording and then released. The $a$ was measured using the photogate until $m_1$ neared the top of the machine. Care was taken to make sure that the weight was dropping the same way for each trial.

The last 2 trials used a different difference in weight between $m_1$ and $m_2$ this was done to try to decrease the error from the first 6 trials by changing more than just $m_1$. Care was taken to make sure that the no damage was done to any equipment due to the increased acceleration.

| Trial | $m_1$ kg | $m_2$ kg | $M$ $\frac{m_2-m_1}{m_1+m_2}$ | $a$ $\mathrm{m \cdot s^{-2}}$ |
|---|---|---|---|---|
| 1 | 0.055 | 0.060 | 0.043 | 0.380 |
| 2 | 0.060 | 0.065 | 0.040 | 0.354 |
| 3 | 0.065 | 0.070 | 0.037 | 0.323 |
| 4 | 0.070 | 0.075 | 0.034 | 0.310 |
| 5 | 0.075 | 0.080 | 0.032 | 0.303 |
| 6 | 0.080 | 0.085 | 0.030 | 0.279 |
| 7 | 0.065 | 0.075 | 0.071 | 0.649 |
| 8 | 0.080 | 0.095 | 0.086 | 0.780 |

Table 1: A table containing all of the values collected during the experiment

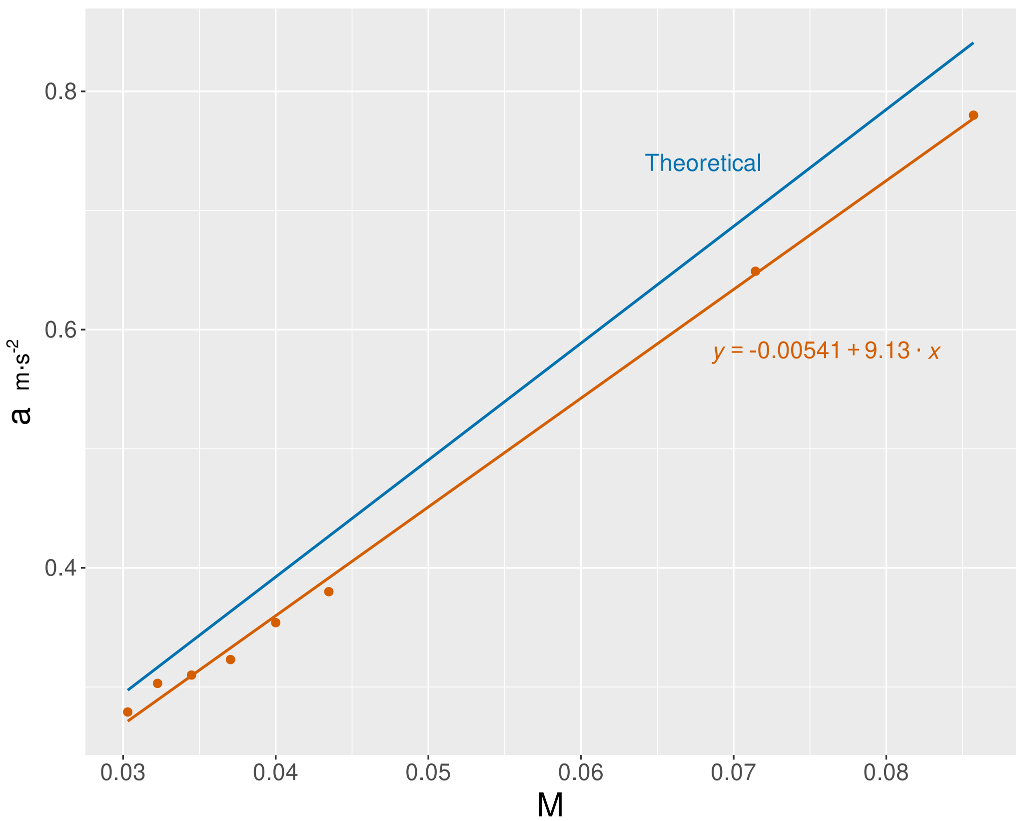



Figure 2: A graphical representation of the line of best fit of $a(M)$ from Table 1 and the theoretical line

## 4 ) Data Analysis

Since the function should yield a linear function with slope $g$, an experimental value for $g$ can be found by finding the line of best fit of the function.

$$g_{\text{experimental}} = 9.13\text{m} \cdot \text{s}^{-2} \quad 4.16$$

Comparing the calculated $g$ to the accepted $g = 9.81\text{m} \cdot \text{s}^{-2}$ the percent deviation can be calculated

$$\%\ \text{deviation} = \left|\frac{T - E}{T}\right| \cdot 100 \quad 4.17$$

$$\%\ \text{deviation} = \left|\frac{9.81 - 9.13}{9.81}\right| \cdot 100 \quad 4.18$$

$$\%\ \text{deviation} = 6.9\% \quad 4.19$$

This error is somewhat high but is still a decent result. The two dominant errors causing this error are systematic and random error. The systematic error can be seen in the graph as the measured acceleration is consistently lower than the theoretical. The primary causes of systematic error were likely friction and air resistance. Although the systematic error has a greater influence on the difference in the theoretical and experimental values of $a$, it does not have as much on the calculated $g$. This is because $g$ is calculated from the slope.

The random error is the primary cause of the error in g and can be seen especially in the lower values of $M$. The primary cause of this random error was inaccuracies in measurement. The readings from the photogate were very inconsistent. Another possible source of random error could be differences in the process of releasing the weight. Even with the care taken, it was likely at least a minor source of error in the experiment. Both errors included here are somewhat difficult to reduce given that they would require upgraded or new equipment. However, the influence of the random error could be reduced by performing more trials.

## 5 ) Conclusion

An Atwood Machine can be a good method for determining acceleration due to gravity. Although experimental errors caused a rather large error of 6.9%, it is still a relatively good approximation. The results could likely be improved by running more trials to decrease the influence of random error. Other methods could be used to decrease error but would likely lead to a much higher complexity and the need for new measurement equipment.