Atwood Machine Lab Report

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Lab Section 12

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1. Introduction

An Atwood's machine shows the relationship between forces and acceleration. It consists of two weights connected by a string. By changing the mass of these weights, the acceleration can be measured. Analyzing the measured accelerations and weights used allows for an experimental measurement of gravity.

2. Theory

The forces in an Atwoods machine can be modeled by drawing a free body diagram for each weight (m_1 and m_2).

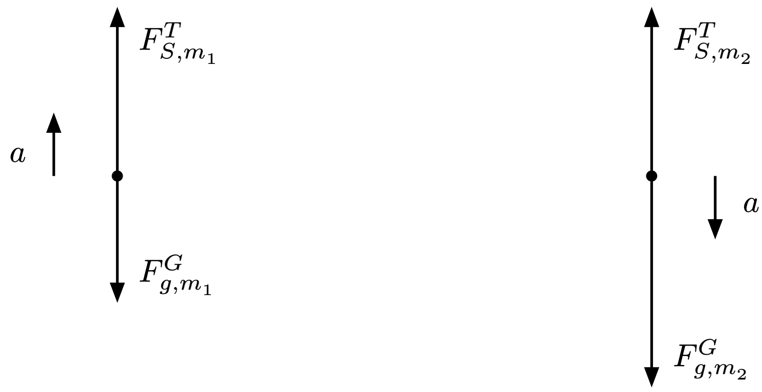

Figure 1: The free body diagrams for m_1 and m_2 where $m_2 > m_1$

The sum of forces in the y direction for each weight can be found by adding the two forces in each diagram.

$$\sum F_{y,m_1} = F^T_{S,m_1} + F^G_{g,m_1} \tag{2.1}$$

$$\sum F_{y,m_2} = F^T_{S,m_2} + F^G_{g,m_2} \tag{2.2}$$

Taking the downward direction to be positive, F^G_{g,m_1} and F^G_{g,m_2} can be found with the equation:

$$F = ma \tag{2.3}$$

$$F^G_{g,m_1} = m_1 g \tag{2.4}$$

$$F^G_{g,m_2} = m_2 g \tag{2.5}$$

Since the string is not stretching, m_1 and m_2 are each exerting equal forces on the string

$$F^T_{m_1,S} = F^T_{m_2,S} = F^T_{m,S} \tag{2.6}$$

Since the tension force acting on the weight and the force that the weight exerts on the string is a force pair, the forces by the string acting on the weights can be found:

$$F^T_{S,m} = F^T_{m,S} \tag{2.7}$$

Using the values found in Equation (2.4), Equation (2.5), Equation (2.7), the equations can be simplified to:

$$\sum F_{y,m_1} = F^T_{S,m} + m_1 g \tag{2.8}$$

$$\sum F_{y,m_2} = F^T_{S,m} + m_2 g \tag{2.9}$$

Using $\sum F_y = m a_y$, the forces can now be related to the weights' accelerations

$$m_1 a_{y,m_1} = F^T_S + m_1 g \tag{2.10}$$

$$m_2 a_{y,m_2} = F^T_S + m_2 g \tag{2.11}$$

Given that the string is still not stretching and that the weights' masses are not the same, the acceleration of the two weights should be equal in magnitude but opposite in direction

$$-m_1 a_y = F^T_S + m_1 g \tag{2.12}$$

$$m_2 a_y = F^T_S + m_2 g \tag{2.13}$$

Equation (2.12) can now be solved for F^T and can be plugged into Equation (2.13)

$$F^T = -m_1 a_y - m_1 g \tag{2.14}$$

$$-m_1 a_y = m_2 g + \left(-m_1 a_y - m_1 g\right) \tag{2.15}$$

$$m_2 a_y = m_2 g - m_1 a_y - m_1 g \tag{2.16}$$

Isolating a then gives an equation for acceleration in terms of m_1 and m_2

$$m_1 a_y + m_2 a_y = m_2 g - m_1 g \tag{2.17}$$

$$a_y = \frac{m_2 g - m_1 g}{m_2 + m_1} \tag{2.18}$$

Pulling g out of the right side of the equation gives

$$a_y = g\left(\frac{m_2 - m_1}{m_1 + m_2}\right) \tag{2.19}$$

Using M to represent $\frac{m_2 - m_1}{m_1 + m_2}$, the equation used for this procedure is found:

$$a_y = gM \tag{2.20}$$

This equation will be used in the procedure using M as the independent variable, and a as the dependent variable to represent the theoretical line. This equation can be used to find the accuracy of the results.

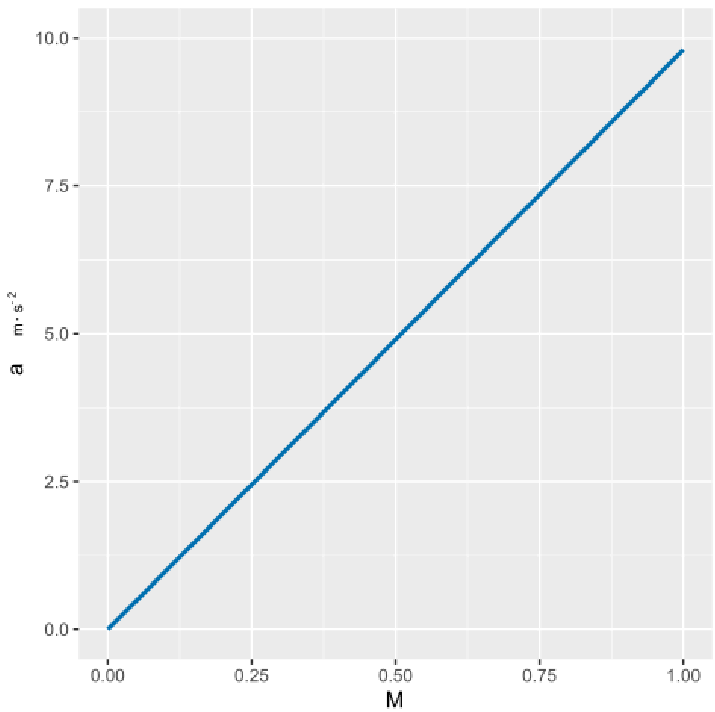

Figure 2: A sketch of the theoretical line for $a(M)$

3. Procedure

An Atwood Machine was created by suspending a string from a wheel attached to a lab support. A photogate was set up so that it was blocked multiple times as the wheel spun. On each end of the string, weights were attached of varying masses.

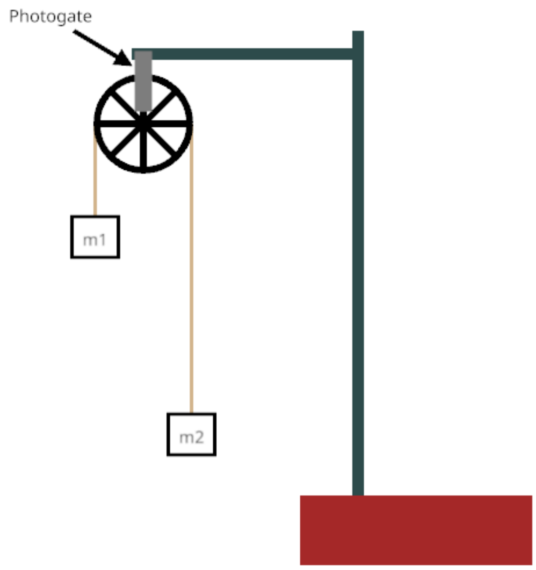

Figure 3: A model of the Atwood machine used in the procedure

The experiment consisted of 8 trials. The first 6 trials were calculated with varying weights for m_1 and $m_2 = m_1 + 0.005$kg The value of M was calculated for each trial The weight was held up until the PASCO Capstone software was recording and then released. The a was measured using the photogate until m_1 neared the top of the machine. Care was taken to make sure that the weight was dropping the same way for each trial.

The last 2 trials used a different change in weight between m_1 and m_2 this was done to try to decrease the error from the first 6 trials by calculating with values of M greater than in the first trials. Care was taken to make sure that the no damage was done to any equipment due to the increased acceleration. The acceleration data was collected from the PASCO Capstone software and written down for later use. M was then calculated from the masses of the weights used for the trial.

Trial	m_1 kg	m_2 kg	M $\frac{m_2-m_1}{m_1+m_2}$	a $\mathrm{m \cdot s^{-2}}$
1	0.055	0.060	0.043	0.380
2	0.060	0.065	0.040	0.354
3	0.065	0.070	0.037	0.323
4	0.070	0.075	0.034	0.310
5	0.075	0.080	0.032	0.303
6	0.080	0.085	0.030	0.279
7	0.065	0.075	0.071	0.649
8	0.080	0.095	0.086	0.780

Table 1: A table containing all of the values collected during the experiment

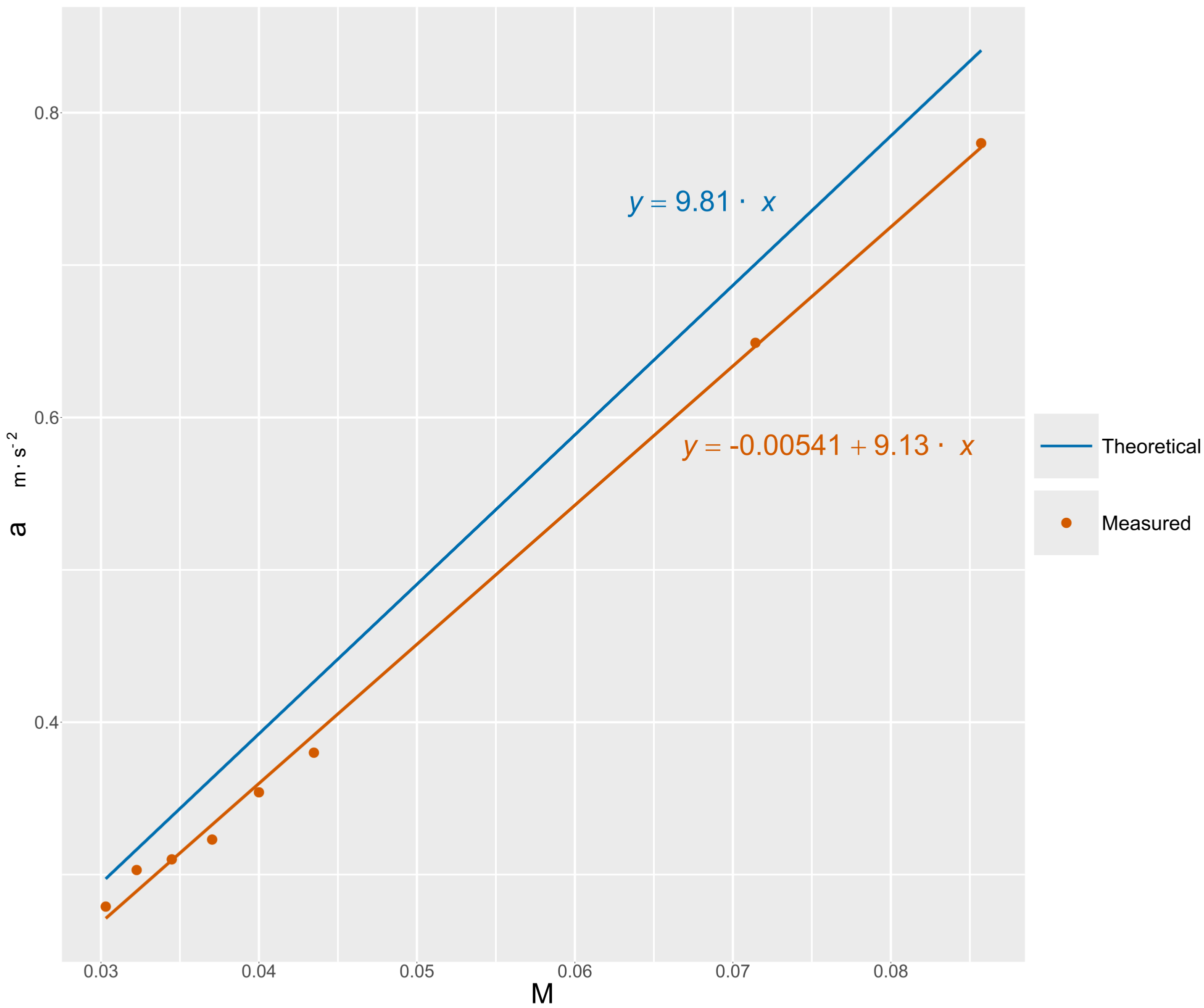

Figure 4: A graphical representation of the measured values, the line of best fit of for the measured values from Table 1, and the theoretical line

4. Data Analysis

Equation (4.20) shows the function should yield a linear function with slope g, an experimental value for g can be found by finding the line of best fit of the function.

$$g_{\text{experimental}} = 9.13\text{m} \cdot \text{s}^{-2} \tag{4.1}$$

Comparing the calculated g to the accepted $g = 9.81\text{m} \cdot \text{s}^{-2}$ the percent deviation can be calculated

$$\%\ \text{deviation} = \left|\frac{T - E}{T}\right| \cdot 100 \tag{4.2}$$

$$\%\ \text{deviation} = \left|\frac{9.81 - 9.13}{9.81}\right| \cdot 100 \tag{4.3}$$

$$\%\ \text{deviation} = 6.9\% \tag{4.4}$$

This error is relatively high but is still a reasonable result. Two causes of error are air resistance and friction. Both are systematic errors that cause the calculated g to be lower than the theoretical g. The systematic error can be seen in the graph by the decreased slope compared to the theoretical line. However, the decreased slope could also be caused by the random error. Both errors are very difficult to remove completely. The best way to improve these errors would be to account for them in the calculations. However, this would increase the complexity of the procedure exponentially as accounting for it would require many more measurements to find the friction and air resistance. Another possible method would be to use more specialized equipment. The error due to air resistance could be almost completely removed by running the experiment in a vacuum. The error due to friction could be reduced by using a more efficient bearing to allow the wheel to turn.

Another likely dominant cause of error is measurement inaccuracy. The photogate was reading very inconsistent values and trials often had to be rerun before they yielded a usable result. The measurement inaccuracy is mostly random error. If there is systematic error caused by it, there are not enough trials to know in which direction it skewed the results. This error is also hard to remove completely, but could easily be improved by running more trials. This would decrease the random error as running more trials will bring the result closer to the average. Another simple but expensive way to decrease this error would be to use better equipment.

5. <u>Conclusion</u>

An Atwood Machine can be a good method for determining acceleration due to gravity. Although experimental errors caused a rather large error of 6.9%, it is still a reasonable approximation. The results could likely be improved by running more trials to decrease the influence of random error. Other methods could be used to decrease error but would likely lead to a much higher complexity, the need for new measurement equipment, or both.